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The calculation of transpiration

by N Thompson

1. Introduction

Consider a crop covering the ground. Let the net radiation at the top of the crop be R_N and the soil heat flux be G. If H and λE are the sensible and latent heat fluxes at the top of the canopy then (ignoring heat storage and photosynthesis)

$$R_N - G = H + \lambda E \qquad (1)$$

Let $T(o)$ be the mean surface temperature of the canopy

Let $q(o)$ be the mean specific humidity in the canopy

Let r_a be the mean (bulk) aerodynamic resistance of canopy (subscripts H and E)

Let r_s be the mean (bulk) stomatal resistance of canopy

Then (with usual notation)

$$(r_a)_H = \varrho c_p \, (T(o) - T(z))/H \qquad (2)$$

$$(r_a)_E = \varrho \, (q(o) - q(z))/E \qquad (3)$$

$$r_s = \varrho \, (q_w \, (T=T(o)) - q(o))/E \qquad (4)$$

(where q_w is saturation specific humidity at temperature T)

Thus $$r_s + (r_a)_E = \varrho \, (q_w \, (T=T(o)) - q(z))/E \qquad (5)$$

Now $$q_w \, (T=T(o)) = \Delta' \, (T(o) - T(z)) + q_w \, (T=T(z)) \qquad (6)$$

where $\Delta' = \partial q_w / \partial T$

Also $$\lambda E = R_N - G - \varrho c_p \, (T(o) - T(z))/(r_a)_H \qquad (7)$$

and $$r_s + (r_a)_E = \varrho \left(\Delta' \, (T(o) - T(z)) + (q_w \, (T=T(z)) - q(z)) \right)/E \qquad (8)$$

$$\therefore \lambda E = R_N - G - \frac{\varrho c_p}{(r_a)_H} \left(\frac{E\,(r_s + (r_a)_E)}{\varrho \Delta'} - \frac{(q_w \, (T=T(z)) - q(z))}{\Delta'} \right) \qquad (9)$$

Writing $$\delta q = q_w \, (T=T(z)) - q(z)$$

Then $$\lambda E = \frac{\Delta' (R_N - G) + \varrho c_p \, \delta q / (r_a)_H}{\Delta' + \frac{c_p}{\lambda (r_a)_H} \, (r_s + (r_a)_E)} \qquad (10)$$

If we make the reasonable assumption that $(r_a)_H = (r_a)_E = r_a$ then

$$\lambda E = \frac{\Delta' (R_N - G) + \varrho c_p \, \delta q / r_a}{\Delta' + \frac{c_p}{\lambda} \, (1 + r_s / r_a)} \qquad (11)$$

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Equation (11) is the usual combination formula employed in the estimation of water losses from vegetated surfaces. In view of how widely it has been exploited it is worth noting some of the assumptions inherent in its derivation:- (a) aerodynamic and stomatal resistances have been represented by mean resistances for the whole canopy, (b) sensible and latent heat fluxes from the underlying soil surface have been included in the canopy fluxes despitebeing controlled by quite different mechanisms.

The wind profile above a uniform rough surface is given by

$$u(z) = \frac{u_*}{k}\left(\ln\left(\frac{z-d}{z_o}\right) - \psi_M\left(\frac{z-d}{L}\right)\right) \qquad (12)$$

z_o is the roughness length of the surface, given approximately by $0.1h$ where h is the height of the vegetation. The zero-plane displacement height d is about $0.63h$ (Monteith 1973) for vegetation such as cereals but may be as large as $0.76h$ for coniferous forest (Thom et al 1975). L (the Monin-Oboukhov length) is defined as

$$L = -u_*^3 T \rho c_p / kgH \qquad (13)$$

From measurements over short vegetation in stable conditions Webb (1970) obtained the expression

$$\psi_M\left(\frac{z-d}{L}\right) = -5.2\left(\frac{z-d}{L}\right) \qquad (14)$$

In unstable conditions, again over short vegetation, the expression

$$\psi_M\left(\frac{z-d}{L}\right) = 2\ln\left(\frac{1+y}{2}\right) + \ln\left(\frac{1+y^2}{2}\right) - 2\tan^{-1}y + \pi/2 \qquad (15)$$

where

$$y = \left(1 - 16\left(\frac{z-d}{L}\right)\right)^{1/4} \qquad (16)$$

(Paulson 1970) has been found to fit the experimental data closely. Over coniferous forests with large values for d and z_o the expressions (14) and (15) may not be applicable (Thom et al 1975, Garrett 1978, Thomson 1978) but suitable alternatives have not yet emerged.

Potential temperature and specific humidity profiles over vegetation in stable conditions may be represented by

$$\theta(z) - \theta(0) = \frac{T_*}{k}\left(\ln\left(\frac{z-d}{z_T}\right) + 5.2\left(\frac{z-d}{L}\right)\right) \qquad (17)$$

$$q(z) - q(0) = \frac{q_*}{k}\left(\ln\left(\frac{z-d}{z_q}\right) + 5.2\left(\frac{z-d}{L}\right)\right) \qquad (18)$$

where

$$T_* = -H/\rho c_p u_* \qquad (19)$$

$$q_* = -E/\rho u_* \qquad (20)$$

(Webb 1970). In unstable conditions the accepted approximations are

$$\theta(z) - \theta(0) = \frac{T_*}{k}\left[\ln\left((1-y_1)/(1+y_1)\right)\right]_{z_T}^{z} \qquad (21)$$

$$q(z)-q(0) = \frac{q_*}{k}\left[\ln\left((1-y_1)/(1+y_1)\right)\right]_{z_q}^{z} \qquad (22)$$

where

$$y_1 = \left(1 - 16\left(\frac{z-d}{L}\right)\right)^{1/2} \qquad (23)$$

(Paulson 1970). z_T and z_q are the roughness lengths for heat and water vapour transfer respectively. These are often assumed to be equal to z_0 but a more plausible relation between them is

$$z_T = z_q = 0.2\, z_0 \qquad (24)$$

(Thom 1975).

If the small differences of actual and potential temperature are ignored over the height range of interest (0–10 m) then Eq. (2) and (21) or (3) and (22) may be combined to give

$$(r_a)_H = (r_a)_E = \frac{1}{k u_*}\left[\ln\left((1-y_1)/(1+y_1)\right)\right]_{z_{T,q}}^{z} \qquad (25)$$

Using Eq (12) to eliminate u_* , one obtains

$$r_a = (r_a)_{H,E} = \frac{1}{k^2 u(z)}\left[\ln\left((1-y_1)/(1+y_1)\right)\right]_{z_{T,q}}^{z} \cdot \left(\ln\left(\frac{z-d}{z_0}\right) - \psi_M\left(\frac{z-d}{L}\right)\right) \qquad (26)$$

Practical use of the combination formula Eq (11) has been restricted usually to periods in excess of 24 hours long. While this may ease some of the difficulties experienced in application of the equation over much shorter periods, it also introduces problems which have not always been given sufficient recognition. These and some other points connected with the use of the equation will be considered now.

2. Some problems in the practical application of the combination formula

(a) The estimation of available energy

There are several published schemes for estimating net radiation at ground level at different times of the day and year, with different cloud amounts.

The current Meteorological Office model (MORECS) deals only with evaporation over periods at least one day long. The relevant radiation equation it uses is

$$R_N = R_a(1-\alpha)(0.18+0.55\,n/N') - 0.95\sigma T_a^4(0.56-0.08e^{1/2})(0.1+0.9\,n/N') \qquad (27)$$

where R_a = daily shortwave radiation for completely transparent sky

α = albedo

n = measured duration of bright sunshine

N' = maximum possible duration of bright sunshine

σT_a^4 = black body radiation at screen temperature T_a °K

e = vapour pressure in mb

The part of the expression giving net long wave radiation was adapted from Brunt (1952), but Swinbank (1963) has noted that the coefficient values vary considerably with the data sets used to derive them: in particular he shows that inclusion of a term involving vapour pressure is not justified by the results. From Benson data Swinbank proposed that the downward component of long wave radiation (Wm^{-2}) may be represented in cloudless conditions by

$$(R_L)_d = -113 + 1{\cdot}03\,\sigma T_a^4 \qquad (28)$$

The net long wave radiation in the absence of cloud is then

$$(R_L)_N = -113 + \sigma\,(1{\cdot}03 T_a^4 - (1-\alpha_L)\,T_s^4) \qquad (29)$$

where T_s = surface temperature

α_L = long wave albedo

σ = 5.67×10^{-8} Wm^{-2} $(°K)^{-4}$

Under clear skies at night typical values for $(T_a - T_s)$ are up to about 6C for short grass, and 3C for bare soil and a tall crop: daytime values may reach around -20C for bare soil, -10C for short grass and -2C for a tall, well-watered green crop (Monteith and Szeicz 1962). Table 1 lists calculated values for $(R_L)_N$ for screen temperatures of 10, 20 and 30C, assuming the values for $(T_a - T_s)$ are those just given: the long wave emissivity of the grass (and taller crops) is assumed to be 0.98 (Gates and Tantraporn 1952) and of bare soil 0.9 (Brooks 1959) (for wet bare soil emissivity $\simeq$ 0.98 (Conaway and Van Bavel 1967)). Values are also given for cases with strong winds where it is assumed

Table 1. Calculated net long wave radiation under clear skies (Wm^{-2})

	T_a = 10C			20C			30C	
	Day	Night	Strong Wind	Day	Night	Strong Wind	Day	Strong Wind
Bare soil (dry)	-168	-52	-66	-172	-43	-58	-176	-51
Short grass	-148	-65	-95	-150	-59	-92	-154	-89
Tall green crop	-105	-80	-95	-103	-75	-92	-102	-89

that $T_a = T_s$. The large diurnal variations for bare soil and short grass reflect the postulated large daytime heating of these surfaces. For a clear day and night in late March or September ($\simeq$ 12 hours of sun), with a mean 24-hour temperature of 15C the predicted average net long wave radiation is around -100 Wm^{-2}, using Eq. 29. This is to be compared with -98 Wm^{-2} for the MORECS model, for an assumed vapour pressure of 15 mb, which at first sight is a satisfactory measure of agreement. However MORECS by not allowing for diurnal temperature variations will underestimate the magnitude of the net long wave component during the day. On the other hand MORECS in its present form calculates R_N as the difference between downward net short wave radiation and (upward) net long wave, the latter over the full day and night period. This is clearly wrong in the context of transpiration calculations when the subtracted long wave radiation should be accumulated during the daylight period only, but it does tend to compensate for numerical underestimation of $(R_L)_N$ during the daytime. In practice the diurnal variation of stomatal resistance should be included when Eq (11) is used and strictly this precludes

the use of Eq (11) with 24-hourly values of short and long wave radiation.

A convenient method of representing the average effects of surface heating on net radiation involves the use of Monteith and Szeicz's (1961) "heating coefficient" β . The relation for net radiation under clear skies then becomes

$$R_N = \left(\frac{1-\alpha}{1+\beta}\right) S + L_o \qquad (30)$$

where S = incident global short wave radiation

L_o = a long wave radiation term

Both β and L_o are functions of crop type and wind speed. At high speeds surface heating to temperatures significantly above screen values is unlikely to occur and β is then negligably small but in light winds values between 0.1 and 0.4 may be appropriate. In applications requiring more precise short term estimates of net radiation a more useful representation for clear skies is

$$R_N = (1-\alpha) S + \sigma\left(1.03 T_a^4 - (1-\alpha_L) T_s^4\right) - 113 \qquad (31)$$

A rough estimate of T_s can be made with the help of eq (12) - (17). The procedure is to first estimate H approximately from a simplified combination equation such as

$$H = R_N - G - \frac{\Delta'(R_N - G) + \varrho c_p \delta q (1 + 0.54 u(2))/100}{\Delta' + 2.4 c_p/\lambda} \qquad (32)$$

(Thom and Oliver 1977), using rough estimates for G and $(R_L)_N$. G may be approximated by

$$G = a R_N \qquad (33)$$

where a (during the daytime) is assumed to be 0.1 for a moderately tall and fairly dense crop such as a cereal approaching maturity, and 0.3 for bare soil: a for grass is taken as 0.2 (Roach (1955), Monteith (1958), **Utaaker** (1966)). For this initial calculation $(R_L)_N$ may be assumed to be $-100\ Wm^{-2}$. The consequent preliminary value for H allows eq (12) and (13) to be solved iteratively for u_* and L . $(T_a - T_s)$ is then calculated using Eq (21). The procedure appears tedious but may be completed very quickly using a computer.

In cloudy conditions the average net radiation during the day can be described by an equation of the form

$$R_N = R_a (1-\alpha)(0.18 + 0.55 n/N') + \left(\sigma(1.03 T_a^4 - (1-\alpha_L) T_s^4 - 113)(0.1 + 0.9 n/N')\right) \qquad (34)$$

or more simply,

$$R_N = R_a \frac{(1-\alpha)}{(1+\beta)}(0.18 + 0.55 n/N') + L_o (0.1 + 0.9 n/N') \qquad (35)$$

where $L_o \simeq -60 Wm^{-2}$ gives the best daytime regression for crop - covered soils (Monteith and Szeicz 1962). The exact values of the short wave coefficients (given as 0.18, 0.55 in Eq (34) and (35)) are determined by the average distribution of cloudiness during the day. Summer mornings are usually more cloud-free than the afternoons in convective conditions, but the diurnal variations in cloud will vary from site to site, for example according to distance from the coast for sites within 20 Km or so of the coast. The short wave coefficients will vary with site therefore: for

example the values given above (obtained from Kew data from 1964-1970) may be compared with 0.14 and 0.80 obtained for the same years at Eskdalemuir. Additionally the values for individual days at a particular site will equal the values obtained from long term averages only when the general type and distribution of cloud on that day is "average", clearly an impossibility.

For short term use (say hourly) an alternative formulation is required. A general equation of the required form is

$$R_N = \frac{(1-\alpha)}{(1+\beta)} S F_1 \sin\theta + L_o F_2 \qquad (36)$$

where θ = solar elevation

S = solar constant (1396 Wm^{-2})

F_1 = transmitted fraction of short wave radiation

F_2 = transmitted fraction of long wave radiation

Now, ignoring refraction

$$\sin\theta = \sin\delta \sin\phi - \cos\delta \cos\phi \cos(\pi t/12) \qquad (37)$$

and

$$\delta = 23.5 \cos\left(2\pi \left(\frac{d'-173}{365}\right)\right) \qquad (38)$$

where ϕ = latitude

t = hour (local time)

δ = solar declination

d' = day of year (1 Jan ≡ 1)

Gadd and Keers (1970) suggest the relations

$$F_1 = (0.6+0.2\sin\theta)(1-m_\ell C_\ell)(1-m_m C_m)(1-m_h C_h) \qquad (39)$$

$$F_2 = (1-n_\ell C_\ell - n_m C_m - n_h C_h) \qquad (40)$$

where $m_\ell = 0.7$, $m_m = 0.7$, $m_h = 0.4$

$n_\ell = 0.6$, $n_m = 0.3$, $n_h = 0.1$

(Kondrateyev 1956, Robinson 1958, Monteith 1962, Lumb 1964). The factor $(0.6 + 0.2 \sin\theta)$ was derived by Lumb from data over the sea where aerosol content of the air is low, but appears to be valid for clean rural sites under the influence of polar air masses (e.g. Unsworth and Monteith 1972). The scattering of solar radiation by aerosol and consequent reduction in the amount reaching the surface varies with the type of air mass and with solar elevation. At average solar elevations of 45° the clean rural site would on average receive about 90% of the amount given by Eq (36) and Eq (39) but less at lower solar elevations and with very turbid atmospheres. For example Unsworth and Monteith suggest that in a typical summer continental air mass a clean rural site (no large towns within about 20 miles) would receive about 85% of the solar radiation given by the equation for 45° solar elevation, and about 75% at mean

elevations of 30°. These are very significant attenuations but would be difficult to incorporate in operational models because of the difficulty of specifying in some automatic manner the air mass type. However in the case of short wave models such as in Eq (34) the use of measured bright sunshine provides a reasonably satisfactory correction for attenuation caused by turbidity as well as by cloud. Thus Unsworth and Monteith showed that the duration of bright sunshine (> 200 Wm^{-2} on a plane perpendicular to the direct beam) on clear but turbid days in late March and late June was 70% and 75% respectively of that expected for a non-turbid atmosphere. Eq (34) gives in these circumstances total radiation 78-81% of that for the completely clean atmosphere, reasonably consistent with the figures given above for a rural site affected by a turbid continental air mass.

A disadvantage of Eq (36) is the observed variation of β with wind speed, particularly for relatively smooth surfaces, and with surface water content for bare soil. For example a value of 0.1 appears to be appropriate to tall crops not under water stress, with little or no variation with wind speed, but over grass Monteith and Szeicz (1962) found β decreasing from 0.3 to 0.15 for a speed increase from 1 to 4 ms^{-1}. Over bare very dry soil β is around 0.4 but decreases to 0.15 to 0.2 for damp soils: presumably some decrease with wind speed is to be expected, particularly for the dry surface. Tall crops under water stress and hence not transpiring freely are warmer by day than well-watered crops and β rises to 0.15 to 0.2 in their case, with an increased dependence on wind speed.

Under clear skies and putting $L_o = -60$ Wm^{-2}, Eq (36) gives for a tall and freely transpiring crop with an average solar elevation of 45 degrees,

$$R_N \simeq 0.6 \times \text{transmitted solar radiation}$$

This agrees well with average values over wheat on summer days of 0.62 at midday and 0.60 for the mean between 0500 and 1800 local time (Denmead 1968).

At night under clear skies a typical value for L_o over a green crop is around -70 Wm^{-2} (Table 1), with about -60 Wm^{-2} over bare soil. When cloud is present an appropriate working relation is

$$R_N = L_o (1 - n_l C_l - n_m C_m - n_h C_h) \qquad (41)$$

(b) The aerodynamic resistance

Aerodynamic resistances deduced using Eq (26) are given in Table 2(a) for 3 values of z_o and a range of values of u, $R_N - G$ and Bowen ratio $B (= H/\lambda E)$: resistances are given for assumed heights of measurement of the wind speed of 2 m and 10 m. In Table 2(b) values of resistance are given on the assumption that $z_o = z_T$. These latter resistances are those most often used in practical applications of combination formulae. Both Tables show similar features, but of course resistances are larger in the case where $z_{T,q} = 0.2\, z_o$ is assumed. Considering in more detail the results in Table 2(b), points to note are the large variations in resistance with changes of z_o, but the relatively small variations with Bowen ratio. Variations associated with changes of wind speed are large but the effects of changing stability are small at the higher wind speeds. Typical bulk stomatal resistances for unstressed crops are around 30-50 sm^{-1} (see below) and so the partitioning of available energy into sensible and latent heat will be quite sensitive to the size of r_a.

(c) Stomatal resistance

Numerous data from field and laboratory measurements on individual leaves of various crops, and from profiles above crop canopies have demonstrated that stomatal resistance depends on 4 main factors (1) Leaf water potential, (2) Temperature (3) Humidity deficit in the canopy and (4) Incident radiation (e.g. Jarvis 1976). For practical purposes the variations with temperature can probably be ignored over the temperature range usually experienced. The

Table 2(a) Aerodynamic Resistances ($z_{T,q} = 0.2 z_o$) (sm^{-1})

Anemo Height m	z_o m	$H+\lambda E$ Wm^{-2}	B = 0.2			B = 5		
			u (ms^{-1})			u (ms^{-1})		
			2	5	10	2	5	10
2m	.01	10	111	46	24	105	45	23
		50	103	45	23	91	44	23
		200	90	44	23	74	42	22
		500	79	43	23	62	38	22
	.1	10	37	15	8.0	37	15	7.6
		50	36	15	7.7	35	15	7.4
		200	34	15	7.4	30	15	7.4
		500	32	15	7.4	26	14	7.4
10m	.01	10	161	73	37	139	70	36
		50	134	69	37	109	63	36
		200	108	63	36	83	54	33
		500	91	57	34	68	47	31
	.1	10	82	35	18	73	34	18
		50	70	34	18	57	32	17
		200	56	30	17	43	28	17
		500	47	30	17	34	25	16

Table 2(b) Aerodynamic Resistances ($z_{T,q} = z_o$) (sm^{-1})

Anemo Height m	z_o m	$H+\lambda E$ Wm^{-2}	B = 0.2			B = 5		
			u (ms^{-1})			u (ms^{-1})		
			2	5	10	2	5	10
2m	.01	10	84	35	18	79	34	17
		50	78	34	17	67	34	17
		200	66	33	17	52	31	17
		500	56	32	17	42	28	17
	.1	10	21	8.6	4.5	20	8.6	4.3
		50	21	8.6	4.3	20	8.5	4.3
		200	20	8.5	4.3	17	8.4	4.3
		500	18	8.4	4.3	14	8.1	4.3
10m	.01	10	128	59	30	108	56	30
		50	104	56	29	81	50	29
		200	80	50	28	59	42	27
		500	65	44	27	46	36	24
	.1	10	59	26	13	51	25	13
		50	49	25	13	38	23	13
		200	38	23	13	27	20	12
		500	30	21	12	20	17	11

resistance decreases very rapidly with increasing light levels for individual leaves but within canopies the light intensity falls off quickly with decreasing height due to leaf absorption: this results in a less rapid fall-off of bulk resistance with increasing radiation than for individual leaves. It has been found that even the stomata of plants growing in soils at field capacity tend to show premature closure in the afternoon due to increasingly negative leaf water potential and increase of specific humidity deficit (e.g. Jarvis 1976). The build-up of water stress in some plants on warm sunny days is so large that loss of turgor and wilting occur even when growing on well-wilted soils: a good example is sugar beet. With these circumstances in mind it is not easy to relate the concept of "potential" evaporation to practical applications. The definition of potential evaporation (evaporation from a green crop completely covering the ground and growing in soil at field capacity - or alternatively "freely" transpiring) has been variously misinterpreted. For example potential evaporation does <u>not</u> mean evaporation from a full-wetted canopy ($r_s = 0$). It must vary with the stage of development of the plant since even if the crop fully covers the ground the amount of radiation reaching the surface depends on the leaf area index. Consequently, the relative importance of direct evaporation from the soil (for which "r_s" $\simeq 0$ if the soil surface is very wet) decreases as the crop's leaf area increases, but of course $r_s \neq 0$ for the leaves, and so "potential" evaporation for a very dense crop might be expected to be less than for sparser vegetation.

Stomatal resistance has been found to vary with the age of leaves, being least for newly-matured leaves, and larger for those which are developing and senescing (Turner 1974). This introduces further complications in the estimation of bulk stomatal resistance. In applications where it may be necessary to estimate transpiration on very short time scales (1 hour or less), such as modelling the drying of canopies account must be taken of the detailed variations of stomatal resistance. Calculation of potential evaporation over periods of at least a day may be carried out with acceptable accuracy using a mean bulk stomatal resistance. These resistances have been estimated usually on the assumption that $z_T = z_o$, and on this basis a typical value for r_s, for well-watered grass is around 30 sm^{-1} (Szeicz and Long 1969, Monteith 1963). With crops such as cereals or potatoes where surface cover varies markedly through the year and where increasing senescence of the crop has to be considered later in the season the effective bulk resistance will be much more variable, but minimum values are probably similar to those given above (e.g. Grant 1975).

The calculation of actual evaporation from vegetation growing on soil with a significant moisture deficit for which r_s is significantly larger will not be considered here. However the approach usually adopted which is to relate the evaporation to some fraction of the calculated potential value is not very sound because the crop will be hotter and the Bowen ratio therefore larger than for the unstressed crop. As a result measured specific humidity deficits will be different from those which would have been observed over freely-transpiring vegetation. It appears therefore that the wrong "potential" evaporation will be calculated in these circumstances. This has significances for irrigation which will be considered very briefly later.

(d) <u>Applications over surfaces of different roughnesses</u>

The combination formula (Eq (11)) allows explicitly for application to surfaces with different roughness by inclusion of the aerodynamic resistance r_a. However there are a number of points to be borne in mind when applying the formula to different surfaces, which are not immediately obvious from an inspection of the equation. It is convenient to consider first a case where evaporation is to be calculated from two effectively infinite surfaces of different roughness, both under the same synoptic influence. It is assumed that the net radiation is the same for either surface and that wind speed (at 10m or 2m), screen temperature and specific humidity (at 1.2m) are measured over the smoother of the surfaces, assumed to be of short grass with a roughness length of 0.01 m.

Eq (32) may then be used to give rough estimates for H and λE. The latent heat flux may be several times larger than H if the crop is well supplied with water and the effects of the water vapour flux on buoyancy must therefore be included in the stability parameter L which is written more exactly as

$$L = -u_*^3 T \rho c_p / k g (H + 0.07 \lambda E) \qquad (42)$$

(this more exact value for L was used in the calculations described in §2(b)). An iterative solution of the wind profile equation (12) leads to values for u_* and L. Equation (26) may then be used to calculate r_a. For consistency the specific humidity deficit should be measured at the height up to which the aerodynamic resistance is calculated. If the height is assumed to be either 2m or 10m then the following corrections (Table 3(a), (b)) should be made to the screen value of the specific humidity deficit before it is entered in the combination formula. The results are given for z_o = 0.01 m

Table 3(a) Corrections to be added to measured specific humidity deficit for use with wind data at 10m ($\times 10^{-3}$)

T deg C	H+λE Wm^{-2}	B = 0.2			B = 5		
		u = 2	5	10 ms^{-1}	2	5	10
5	10	.04	.01	-.01	-.08	-.06	-.05
	50	.21	.15	.07	-.15	-.15	-.11
	200	.53	.54	.36	-.30	-.36	-.31
	500	.92	1.08	.86	-.48	-.61	-.61
15	10	0	-.03	-.04	-.15	-.13	-.10
	50	.13	.09	.02	-.32	-.32	-.24
	200	.39	.40	.26	-.64	-.76	-.66
	500	.71	.84	.66	-1.02	-1.30	-1.30
25	10	-.06	-.08	-.09	-.27	-.23	-.17
	50	.02	-.01	-.05	-.57	-.58	-.42
	200	.19	.19	.10	-1.15	-1.37	-1.20
	500	.39	.47	.36	-1.86	-2.35	-2.35

but will be smaller for rougher surfaces because of increased turbulent mixing. The corrections, listed for two widely different Bowen ratios, are large at high temperatures and where the latent heat flux is relatively small. However the corrections are near-zero for B = 0.5 so for many applications, particularly when used with wind speeds at 2m, they may be assumed negligible.

Eq.(12) may be solved iteratively to provide factors converting wind speed measured at 10m to estimates at 2m. The results are given in Table 4 for three different surface roughnesses.

Table 3(b) Corrections to be added to measured specific humidity deficit for use with wind data at 2m ($\times 10^3$)

T deg C	H+λE Wm^{-2}	B = 0.2			B = 5		
		u = 2	5	10 ms^{-1}	2	5	10
5	10	.02	.01	0	-.02	-.01	-.01
	50	.07	.05	.01	-.04	-.02	-.02
	200	.19	.17	.10	-.10	-.11	-.08
	500	.33	.35	.24	-.16	-.20	-.17
15	10	.01	0	0	-.04	-.02	.01
	50	.06	.03	.02	-.09	-.08	-.01
	200	.15	.13	.08	-.21	-.23	-.17
	500	.27	.28	.19	-.35	-.42	-.37
25	10	0	0	-.01	-.06	-.04	-.02
	50	.07	.02	0	-.17	-.15	-.09
	200	.09	.08	.04	-.38	-.42	-.31
	500	.16	.17	.12	-.63	-.77	-.68

Table 4. Factors converting wind speed at 10 m to speed at 2 m

z_o m	H+λE Wm^{-2}	B = 0.2			B = 5		
		u = 2	5	10 ms^{-1}	2	5	10
.01	10	.76	.76	.76	.78	.76	.76
	50	.77	.76	.76	.84	.80	.77
	200	.83	.80	.77	.86	.82	.78
	500	.85	.81	.78	.87	.84	.80
.03	10	.71	.71	.71	.73	.71	.71
	50	.72	.71	.71	.79	.73	.71
	200	.78	.73	.71	.80	.76	.72
	500	.80	.75	.72	.81	.78	.74
.10	10	.58	.58	.58	.59	.58	.58
	50	.60	.58	.58	.64	.59	.58
	200	.64	.59	.58	.66	.61	.59
	500	.65	.60	.58	.67	.63	.60

In a given synoptic situation the wind speeds near the surface will not be equal over the rough and smoother surfaces under consideration. The Monin-Oboukhov similarity theory which was developed originally for the surface layer and used already in this memorandum has been extended to the whole boundary layer (e.g. Zilitintsevitch et al 1967), and more recently second-order closure modelling has been used to describe the behaviour of wind and temperature profiles in the convective boundary layer. For the present purpose the results of a similarity treatment given by Wyngaard et al (1974) and adapted to fit the results of the "Wangara" experiment (Clarke et al 1971) will be used. The original data showed considerable scatter but may be fitted approximately by a relation of the form

$$\frac{G_0}{u_*} = 4 \ln\left(\frac{-L}{6 z_0}\right) \qquad (43)$$

where G_0 is the surface geostrophic wind. This equation breaks down in near-neutral conditions: in this situation Arya (1975) presents data fitting the relation

$$\frac{G_0}{u_*} = 2.2 \ln\left(\frac{12000\, G_0}{z_0}\right) \qquad (44)$$

Equation 43 cannot be solved directly because of the interaction of L, z_0 and u_*, but the required results may be obtained readily by iteration, using also some equations given earlier. Wind speeds at 10m calculated in this way are presented in Table 5 for z_0 values of 0.01 and 0.1m (corresponding to short grass, and mature cereals) for various surface buoyancy fluxes and geostrophic winds. The rate of extraction of momentum from air within the boundary layer is very roughly proportional to the square of the surface wind: hence with high geostrophic winds so much momentum is being removed at the surface that even the enhanced turbulence mixing in the boundary layer (roughly proportional to surface wind to the first power) is unable to bring down momentum from

Table 5 10 m winds for different geostrophic winds

G_0 ms^{-1}	z_0 m	$H + \lambda E$ Wm^{-2}	$u(10)$ (ms^{-1}) B = 0.2	1	5
3	.01	10	1.80(1.56)	1.90	1.97
		50	2.00	2.12	2.20
		200	2.20	2.34	2.41
		500	2.35	2.50	2.59
	.10	10	1.54(1.23)	1.66	1.73
		50	1.79	1.93	2.02
		200	2.03	2.21	2.31
		500	2.22	2.42	2.54
5	.01	10	– (2.52)	2.85	2.95
		50	3.00	3.19	3.28
		200	3.30	3.50	3.61
		500	3.50	3.72	3.85
	.10	10	– (1.94)	2.40	2.51
		50	2.57	2.79	2.92
		200	2.93	3.17	3.31
		500	3.19	3.45	3.62
10	.01	10	– (4.80)	–	–
		50	5.13	5.47	5.67
		200	5.69	6.06	6.26
		500	6.08	6.46	6.66
	.10	10	– (3.69)	–	–
		50	4.24	4.60	4.80
		200	4.82	5.23	5.46
		500	5.25	5.69	5.94
20	.01	10	– (9.23)	–	–
		50	–	–	9.71
		200	9.75	10.41	10.77
		500	10.45	11.15	11.51
	.10	10	– (7.03)	–	–
		50	–	–	7.90
		200	7.95	8.61	8.98
		500	8.65	9.38	9.79

Note: Missing values correspond to breakdown of Eq. 43 in near-neutral conditions
Bracketed values were obtained from Eq.44 for neutral conditions.

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aloft fast enough to compensate for this. Thus the ratio of surface to geostrophic winds decreases as G_o increases. At geostrophic speeds of 3 and 20 ms^{-1} at the highest instabilities ($\beta = 5$, $H+\lambda E = 500\ Wm^{-2}$) the ratios of winds over the smoother and rougher surfaces are 1.02 and 1.18, reflecting the much-reduced importance of buoyancy-induced mixing at high wind speeds.

The profiles of potential temperature and specific humidity will also differ over the two surfaces due to the differing u_*'s , and consequently the specific humidity deficit and temperature at the reference height also vary with surface roughness. Thus evaporation calculated on the basis of similar humidity deficits over the two surfaces will be incorrect (e.g. as in Ag.Me.823) but the calculation of the change of humidity deficit with change of underlying surface is extremely complicated because of the number of interacting processes which are involved: it will not be attempted here. In any case empirical expressions such as Eq.43 but now for T_* and q_* which are needed to solve the problem are less well defined as yet than those for u_* (particularly in the case of q_*) and further effort in this direction is probably not justified until more results appear in the literature.

In many practical situations the standard measurements of wind, temperature and humidity deficit are made over a relatively smooth site, but it may be necessary to use these values to estimate evaporation over a rougher field nearby. The airflow over the field will not be in equilibrium with the whole boundary flow and equations such as (43) may not be used to estimate the change of wind speed and hence u_* which occurs as air flows from the smoother to the rougher surface. There are numerous theoretical treatments which provide estimates for the changes which take place. Mulhearn (1977) presents an approach which gives estimates for changes in u_* in near-neutral conditions which agree closely with limited experimental data. His results show that for flow over a smooth-to-rough transition with z_o values of respectively 0.01 and 0.1 m then the ratio of u_* over the two surfaces is 1.25 at 100 m downwind of the roughness change and 1.18 at 1000 m downwind. The theory shows the ratio continuing to decrease as distance downwind increases since it takes no account of adjustment throughout the boundary layer. At large fetches the limiting value of the u_* ratio is 1.18 (from Eq.44 for a geostrophic wind of 10 ms^{-1}) and it is probable therefore that the ratio falls below this limiting value at intermediate fetches.

In non-neutral conditions the problem is much more complex because of the interaction between u_* , T_* and q_* . A satisfactory treatment does not appear to have been worked out yet. It is clear that changes in u_* as the air flows over the rougher surface will be accompanied by changes of wind speed, temperature and specific humidity deficit but it is not worth speculating at this stage on the magnitude of the changes in evaporation which will occur.

(e) Evaporation at night

At night the stomata are closed and r_s must then be replaced by a bulk cuticular resistance r_c which is typically at least an order of magnitude larger than r_s . Transpiration then is negligible but evaporation can continue from the soil surface, though this is likely to be small for a dense canopy. Condensation may occur if winds are very light, at a rate given by putting $r_s = 0$, which corresponds to a relative humidity of 100% within the canopy.

(f) Evaporation of intercepted rain

When the canopy foliage is wet the combination formula becomes

$$\lambda E = \frac{\Delta' (R_N - G) + \varrho c_p \delta q / r_a}{\Delta' + c_p / \lambda} \qquad (45)$$

c_p/λ is about 0.4×10^{-3} and Δ' is around 0.9×10^{-3} at typical daytime summer temperatures of 20 deg C. The majority of the available energy will be used in evaporation and if the winds are not too light the stability will be close to neutral. In these circumstances the aerodynamic evaporation is directly proportional to wind speed and therefore substantially larger than transpiration due to the aerodynamic term in the case of a dry canopy, particularly at higher speeds. The ratio of evaporation of intercepted water to transpiration, at 20 deg C for r_s = 30 sm^{-1} is

$$\lambda E(\text{intercepted})/\lambda E(\text{transpiration}) = (1.3 + 12/r_a)/1.3 \qquad (46)$$

Some values for the ratio (i) for a fairly smooth surface (z_o = 0.01 m) with 10 m wind speeds of 2 and 10 ms^{-1} and (ii) for a rougher surface (z_o = 0.1 m) at the same wind speeds are respectively (i) 1.07, 1.31 and (ii) 1.15, 1.71. The rate of evaporation therefore becomes markedly larger than the potential rate over rough surfaces in windy conditions. At typical winter temperatures ($T \simeq 5C$) the ratio of these "aerodynamic" evaporation rates becomes even larger, increasing for example to 2.18 for the rougher surface in winds of 10 ms^{-1}.

(g) Advection and the calculation of irrigation need

Philip (1959) showed that the variation of evaporation over an irrigated plot with distance x downwind from a large dry area is given by

$$E = ax^{-p} + b \qquad (47)$$

where $p \simeq 0.17$. The formula was derived for the case when the wind profiles over both the dry and irrigated areas were the same, implying no change of roughness or stability in crossing the boundary. The irrigated plot is of course cooler than the drier surroundings during the day so Eq (47) is only a rough approximation, though it is supported to some extent by some experimental data (Lang et al 1974). Dyer and Crawford (1965) presented results obtained over a field of irrigated grass in dry surroundings which showed that the variation of E with x depended strongly on the magnitude of the available energy $R_N - G$. Where this was large ($\sim$ 400 wm^{-2}) and the wind only moderate (6 ms^{-1} at 10 m) most of the evaporation was sustained directly by the available energy, and any extra energy required was drawn from a very shallow layer of the atmosphere. In this particular case E decreased by only a few per cent in 150 m. However when the energy available from radiation was much less than the observed evaporation rate then a considerable amount of heat had to be drawn from the atmosphere and a substantial inversion built up as distance downwind increased. This inhibited turbulent mixing and the transfer downwards of sensible heat to the surface, and so the evaporation decreased markedly with increasing travel. In one case where the wind speed was 11 ms^{-1} and the available energy was 90 Wm^{-2} the vertical flux of latent heat decreased from about 300 Wm^{-2} at the upwind edge to around 180 Wm^{-2} after 150 m. Davenport and Hudson (1967, a, b) presented data from irrigated cotton plots in an arid region which demonstrated marked reductions (30% or more) in the evaporation rate as the distance downwind from the upwind edge of the plots increased to 300 m. They showed that temperature, specific humidity deficit and wind speed all decreased with increasing downwind distance, which is to be expected because of the increasing vertical stability and the uptake of moisture over the plots. They also described results obtained over a transpiring crop of sugar beet downwind of dry stubble in England

where temperature fell by 1 deg C, and the specific humidity deficit decreased by around 1×10^{-3} after 70 m travel across the beet field. The reduction of evaporation was around 20% over this distance. The authors gave no details of available energy and the results cannot be interpreted therefore in the way that Dyer and Crawford did. There is at least an indication that in "droughty" weather the central and downwind areas of irrigated plots will require less water than near the upwind edge. Near this edge in particular the evaporation may exceed by a substantial amount the potential evaporation calculated from observations made over the generally parched terrain elsewhere.

(i) Comparisons with MORECS (1978)

MORECS uses Thom and Oliver's (1977) modification of the combination formula

$$\lambda E = \frac{\Delta'(R_N - G) + 12 S_q (1 + 0.43 u(10))}{\Delta' + 2.4 c_p/\lambda} \qquad (48)$$

If it is assumed that the crop is dry but well-supplied with water, and that daytime evaporation only is being considered, then Table 6 shows a comparison of the aerodynamic contribution to the evaporation according to Thom and Oliver's

Table 6 A comparison of aerodynamic evaporation calculated by two methods

	r_s sm^{-1}	$u(10)$ ms^{-1}	λE (Wm^{-2})		
			Thom & Oliver	Eq.11 (z_o=0.01)	Eq.11 (z_o=0.1)
T = 8C, T_d = 5C, $H \sim 0$	30	2	19	12	23
		5	32	24	44
		10	53	39	65
	50	2	19	11	21
		5	32	21	33
		10	53	33	48
T = 20C, T_d = 10C, $H \simeq 50 Wm^{-2}$	30	2	81	73	140
		5	138	109	197
		10	232	168	289
	50	2	81	69	123
		5	138	98	165
		10	232	144	225

equation (i.e. the second half of the expression on the RHS of Eq. (48)) and as calculated using the more exact combination formula (Eq. 11): the latter computations were made for surface roughness lengths of 0.01 and 0.1 m, for assumed stomatal resistances of 30 and 50 sm^{-1}. Results are given for three wind speeds, for a "winter" case (T = 8C, T_d = 5C, $H \sim 0$) and a "summer" case (T = 20C, T_d = 10C, $H \simeq 50 Wm^{-2}$). Stomatal resistances in winter are likely to be in excess of the 30 sm^{-1} suggested earlier because of low incident radiation and low temperature and a better estimate may be the alternative value of 50 sm^{-1} given in the Table. Results by each method are then in good agreement if z_o is as large as 0.1 m. This surface roughness is larger than for any winter field crops and it appears likely therefore that Eq (48) overestimates the aerodynamic evaporation in this season. In the example chosen for summer, with its large vapour pressure deficit it is likely that the stomatal resistance will again be larger than 30 sm^{-1}. Selecting 50 sm^{-1} as more probable then differences between evaporation calculated by the two methods show systematic changes as wind speed increases. It appears therefore that a

simplified combination formula which lacks a term in its denominator involving wind speed in some form to represent aerodynamic resistance is not satisfactory.

Neglect of such a term also affects the estimation of the contribution to evaporation from the available energy. Table 7 shows the ratio of this contribution to evaporation calculated by Eq (48) and Eq (11). Selecting values for r_s of 50 sm^{-1} for the reasons given above it is seen that the Thom and Oliver

Table 7 Ratio of contributions to evaporation from available energy calculated by Thom and Oliver's method, and by Eq. (11)

	r_s sm^{-1}	$u_{(10)}$ ms^{-1}	λE (Eq.48) / λE (Eq.11)	
			z_o = 0.01 m	z_o = 0.1 m
T = 8C T_d = 5C H ~ 0	30	2	.68	.75
		5	.76	.94
		10	.89	1.26
	50	2	.72	.84
		5	.85	1.16
		10	1.07	1.69
T = 20C T_d = 10C H ≃ 50Wm^{-2}	30	2	.78	.87
		5	.83	.98
		10	.92	1.20
	50	2	.84	.99
		5	.92	1.17
		10	1.07	1.56

formulation performs unsatisfactorily, because of the substantial changes in this component of evaporation caused by both changes of wind speed and surface roughness. The failings of the Thom and Oliver formula are not surprising when it is recalled that its adjustable constants were obtained from annual estimates of evaporation. Obviously the neglect of variations of aerodynamic resistance as they affect both denominator and numerator of this formula has not been successful.

3. Concluding remarks

This memorandum has discussed various problems in the application of combination-type formula to the estimation of evaporation from fully-vegetated surfaces. It is clear that simplified equations such as that suggested by Thom and Oliver (1977) are not suited to the estimation of potential transpiration except on the annual time scale. Over shorter periods it is necessary to accord proper recognition to the role played by variations in aerodynamic resistance. It is worth reiterating that, on the assumption that the crop cover is dense (with negligible evaporation from the soil surface), combination formulae must be used only for the daytime to calculate transpiration. Average winds are usually stronger by day than at night and account should be taken of this when wind data are given as 24-hour runs. Nighttime applications of the formula to a dense crop must be restricted to the estimation of either the rates of evaporation of intercepted water, or the rate of condensation of water onto the crop, in each case by putting r_s =o.

Any comments or suggestions on this paper should be referred to the author.

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